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THE NUMERICAL STUDY OF EFFORTS FROM A 4R SPHERICAL QUADRILATERAL MECHANISM

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Abstract**—**The 4R spherical quadrilateral mechanism, where by R is notated the rotation cinematic couple, from which is obtained the cardan mechanism, is of third family , as consequence, triple static undetermined and for calculating the reactions from the cinematic couples it is applied the elastic linear calculation using the relative displacements method noted in plukerian coordinates.

By using this method and numerical calculation, allows one to determine the reactions from the cinematic couples, as the method can be applied to different types of spatial or plane mechanisms.

The calculation algorithm established in this paper and the program that was made, allows the user to determine the influence of mechanical and geometrical characteristics each element of the spherical 4R mechanism over the reactions from its cinematic couples.

The results of the numerical solving of this problem will be presented under the form of a diagram and will be commented.

Keywords**—** cardan transmission, elasticity, pluckerian,

I. INTRODUCTION

THE mathematic model for calculating efforts in such system was elaborated in paper [1] and the calculation algorithm is presented in the paper.

The obtained results allow us to conclude the influence of geometrical and mechanical parameters over the unitary efforts that appear in the cinematic couples of the cardan articulation.

II. THEORETICAL ASPECTS

It is considered the 4R spatial spherical mechanism from Fig. 1., in initial position, with the rotation axes concurrent in O, OA, OB, OC, and OD, the axes OB, OC being both perpendicular, with the length of the elements l_i, d_i, elastic constants E and G and the reference systems

- $OX_0Y_0Z_0$, fixed system;
- $OX'Y_0Z_0'$, obtained by rotating the reference system $OX_0Y_0Z_0$, around the axis OY_0 with the angle α ;
- OX_0Y^*Z, obtained by rotating the system $OX_0Y_0Z_0$ around the axis OX_0 with the angel θ_1 ;
- $OX'YZ_0^*$, obtained by rotating the system $OX'Y_0Z_0'$ around the axis OX' with the angle θ_2 ;
- OX^*YZ three-orthogonal system, where the axis OY is perpendicular on OZ;
- $Ox_iy_iz_i$ the local reference systems that contain the elements $l_i, i = 1,2,\ldots 8$.

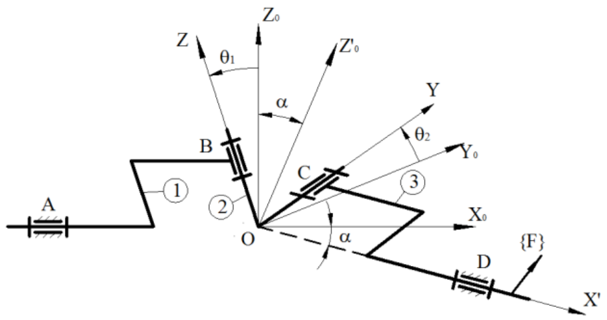

Fig. 1. 4R Asymmetrical spatial spherical mechanism

During the movement, the rotation cinematic couples B, C stay on the fixed axis OZ, OY and the sections AB, BC, CD remain solidar with the reference systems OX_0Y^*Z, OX^*YZ, $OX'YZ_0^*$.

The flexibility matrixes $\left[H_{AB}^*\right]; \left[H_{BC}^*\right]; \left[H_{CD}^*\right]$ reported to the previously defined reference systems remain invariable and are calculated with the relations

$$\begin{gathered}[\mathbf{H_{AB}}]=[\mathbf{H_1}]+[\mathbf{H_2}]+[\mathbf{H_3}] \\ [\mathbf{H_{BC}}]=[\mathbf{H_4}]+[\mathbf{H_5}] \\ [\mathbf{H_{CD}}]=[\mathbf{H_6}]+[\mathbf{H_7}]+[\mathbf{H_8}]\end{gathered} \tag{1}$$

where

$$\left[H_i^*\right]=\left[T_i^*\right]\left[h_i\right]\left[T_i^*\right]^{-1}. \tag{2}$$

$$[h_i]=\begin{bmatrix} 0 & 0 & 0 & \frac{l_i}{GI_{xi}} & 0 & 0 \\ 0 & 0 & \frac{l_i^2}{2EI_{yi}} & 0 & \frac{l_i}{EI_{yi}} & 0 \\ 0 & -\frac{l_i^2}{2EI_{zi}} & 0 & 0 & 0 & \frac{l_i}{EI_{zi}} \\ \frac{l_i}{EA_i} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{l_i^3}{3EI_{zi}} & 0 & 0 & 0 & -\frac{l_i^2}{2EI_{zi}} \\ 0 & 0 & \frac{l_i^3}{3EI_{yi}} & 0 & \frac{l_i^2}{2EI_{yi}} & 0 \end{bmatrix}. \quad (3)$$

$[T_i]$ being the position matrixes of the local reference systems $O_i x_i y_i z_i$ that contains the element l_i, in report with the same systems previously presented, given by the expressions with the form

$$[T_i]=\begin{bmatrix} [R_i] & [0] \\ [G_i][R_i] & [R_i] \end{bmatrix}$$
$$[T_i]^{-1}=\begin{bmatrix} [R_i]^T & 0 \\ [R_i]^T[G_i]^T & [R_i]^T \end{bmatrix}. \quad (4)$$

and $[R_i],[G_i]$ the rotation and translation matrices given by the relations

$$[G_1]=\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & l_1+l_2 \\ 0 & (l_1+l_2) & 0 \end{matrix};[G_2]=\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & l_3 \\ 0 & l_3 & 0 \end{matrix}$$
$$[G_3]=\begin{matrix} 0 & l_3 & 0 \\ l_3 & 0 & l_3 \\ 0 & l_3 & 0 \end{matrix};[G_4]=[G_5]=\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \quad (5)$$
$$[G_6]=\begin{matrix} 0 & 0 & l_5 \\ 0 & 0 & 0 \\ l_5 & 0 & 0 \end{matrix};[G_7]=[G_8]=\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & l_6 \\ 0 & l_6 & 0 \end{matrix}.$$

$$[R_1]=[R_3]=[R_6]=[R_8]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$[R_2]=[R_4]=\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix};[R_5]=\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (6)$$
$$[R_8]=\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}.$$

In relation with the reference system $OX_0Y_0Z_0$, the reliability matrixes are given by the relations

$$[H_{AB}]=[T_{AB}][H^*_{AB}][T_{AB}]^{-1}$$
$$[H_{BC}]=[T_{BC}][H^*_{BC}][T_{BC}]^{-1} \quad (7)$$
$$[H_{CD}]=[T_{CD}][H^*_{CD}][T_{CD}]^{-1}.$$

where with $[T]$ where noted the position matrixes given by the expressions like

$$[T]=\begin{bmatrix} [R] & [0] \\ [0] & [R] \end{bmatrix}$$
$$[T]^{-1}=\begin{bmatrix} [R]^T & [0] \\ [0] & [R]^T \end{bmatrix}. \quad (8)$$

$[R]$ being the rotation matrix

$$[R_{AB}]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta_1 & -s\theta_1 \\ 0 & s\theta_1 & c\theta_1 \end{bmatrix}$$
$$[R_{BC}]=\begin{bmatrix} c\theta_1 c\theta_2 + s\theta_1 s\theta_2 c\alpha & s\alpha s\theta_2 & 0 \\ -c\theta_1 s\theta_2 s\alpha & c\theta_2 & -s\theta_1 \\ -s\theta_1 s\theta_2 s\alpha & c\alpha s\theta_2 & c\theta_1 \end{bmatrix} \quad (9)$$
$$[R_{CD}]=\begin{bmatrix} c\alpha & s\alpha s\theta_2 & s\alpha c\theta_2 \\ 0 & c\theta_2 & -s\theta_2 \\ -s\alpha & c\alpha s\theta_2 & c\alpha c\theta_2 \end{bmatrix}.$$

where by *c*, *s* where noted the trigonometric functions *cos* and *sin*.

To the rotation cinematic couples from B, C, D, [5], [6], are attached the column matrixes of the pluckerian coordinates

$$\{U_B\}=[0,-s\theta_1,c\theta_1,0,0,0]^T$$
$$\{U_C\}=[s\theta_2 s\alpha,c\theta_2,s\theta_2 c\alpha,0,0,0]^T \quad (10)$$
$$\{U_D\}=[c\alpha,0,-s\alpha,0,0,0]^T.$$

$$\{\tilde{U}_B\}=[0,0,0,0,-s\theta_1,c\theta_1]$$
$$\{\tilde{U}_C\}=[0,0,0,s\theta_2 s\alpha,c\theta_2,s\theta_2 c\alpha]^T \quad (11)$$
$$\{\tilde{U}_D\}=[0,0,0,c\alpha,0,-s\alpha]^T.$$

and also the matrixes

$$[U]=[\{U_B\},\{U_C\},\{U_D\}] \quad (12)$$

$$\left[\tilde{U}\right]=\left[\left\{\tilde{U}_B\right\},\left\{\tilde{U}_C\right\},\left\{\tilde{U}_D\right\}\right]^T. \tag{13}$$

If the column matrix of the pluckerian coordinates of the reaction from A is

$$\left\{R_A^0\right\}=\left[R_{AX}^0,R_{AY}^0,R_{AZ}^0,M_{AX}^0,M_{AY}^0,M_{AZ}^0\right]^T. \tag{14}$$

then the cinematic couple from A is blocked and the axis OD is being driven by the moment $\tilde{M}$,then [6]

$$\left[\tilde{U}\right]\left\{R_A^0\right\}=\left[0,0,-\tilde{M}\right]^T. \tag{15}$$

and from here, taking into account the transmission function [2], [4], [7]

$$tg\theta_2=\frac{tg\theta_1}{\cos\alpha}. \tag{16}$$

are obtained the components *statically determined*, in the $OX_0Y_0Z_0$ system of the reaction $\left\{R_A^0\right\}$

$$M_{AX}^0=-\frac{\tilde{M}c\alpha}{s^2\theta_1+c^2\alpha c^2\theta_1}$$
$$M_{AY}^0=\frac{\tilde{M}s\alpha s\theta_1 c\theta_1}{s^2\theta_1+c^2\alpha c^2\theta_1} \tag{17}$$
$$M_{AZ}^0=\frac{\tilde{M}s\alpha s^2\theta_1}{s^2\theta_1+c^2\alpha c^2\theta_1}.$$

For determining the components *statically undetermined* $R_{AX}^0,R_{AY}^0,R_{AZ}^0$, [8], the linear elastic calculation is used and with the help of the relations

$$\begin{gathered}\left[H_{AD}\right]=\left[H_{AB}\right]+\left[H_{BC}\right]+\left[H_{CD}\right]\\ \left[K_{AD}\right]=\left[H_{AD}\right].\end{gathered} \tag{18}$$

notating with $\{\xi\}$ the matrix

$$\{\xi\}=\left[\xi_B,\xi_C,\xi_D\right]^T. \tag{19}$$

where ξ_B,ξ_C,ξ_D are the elastic rotations from the cinematic couples, are obtained [3] consecutive relations

$$\{\xi\}=\left[\left[\tilde{U}\right]\left[K_{AD}\right]\left[U\right]\right]^{-1}\left[0,0,-\tilde{M}\right]^T. \tag{20}$$

$$\left\{R_A^0\right\}=\left[K_{AD}\right]\left[U\right]\{\xi\}. \tag{21}$$

from which are deducted, for values of angle θ_1 from degree to degree, the rotations values ξ_B,ξ_C,ξ_D and the reaction forces $R_{AX}^0,R_{AY}^0,R_{AZ}^0$.

The reactions from the cinematic couples B, C, D, expressed in the general reference system $OX_0Y_0Z_0$ are obtained from the balance equations and taking into account the action-reaction principle the relations are given

$$\begin{gathered}\left\{R_B^0\right\}=\left\{R_C^0\right\}=\left\{R_A^0\right\}\\ \left\{R_D^0\right\}=\left\{R_A^0\right\}+\tilde{M}\left[0,0,0,c\alpha,0,-s\alpha\right]^T.\end{gathered} \tag{22}$$

The physical type of these reactions is perceived by expressing them in the local reference systems through the relations

$$\left\{R_A\right\}=\left[T_A\right]^{-1}\left\{R_A^0\right\} \tag{23}$$

where

$$\begin{gathered}\left[T_A\right]^{-1}=\begin{bmatrix}\left[R_A\right]^T & 0\\ \left[R_A\right]^T\left[G_A\right]^T & \left[R_A\right]^T\end{bmatrix};\left[R_A\right]=\left[R_{AB}\right]\\ \left[G_A\right]=\begin{bmatrix}0 & -Z_A & Y_A\\ Z_A & 0 & -X_A\\ -Y_A & X_A & 0\end{bmatrix}.\end{gathered} \tag{24}$$

$$\begin{gathered}\left[T_B\right]^{-1}=\begin{bmatrix}\left[R_B\right]^T & 0\\ \left[R_B\right]^T\left[G_B\right]^T & \left[R_B\right]^T\end{bmatrix};\left[R_B\right]=\left[R_{BC}\right]\\ \left[G_B\right]=\begin{bmatrix}0 & -Z_B & Y_B\\ Z_B & 0 & -X_B\\ -Y_B & X_B & 0\end{bmatrix}.\end{gathered} \tag{25}$$

$$\begin{gathered}\left[T_C\right]^{1}=\begin{matrix}\left[R_C\right]^T & 0\\ \left[R_C\right]^T\left[G_C\right]^T & \left[R_C\right]^T\end{matrix};\left[R_C\right]=\left[R_{BC}\right]\\ \left[G_C\right]=\begin{matrix}0 & Z_C & Y_C\\ Z_C & 0 & X_C\\ Y_C & X_C & 0\end{matrix}.\end{gathered} \tag{26}$$

III. CALCULATION ALGORITHM

There are considered as known the following geometrical and mechanical characteristics $\alpha,l_i,A_i,I_{Ix},I_{Iy},I_{Iz},E_i,G_i,i=1,2,\ldots,8$

– the following constant matrixes are determined with the relations (1) - (6)

$$\left[H_{AB}\right],\left[H_{BC}\right],\left[H_{CD}\right], \tag{27}$$

the flexibility and position matrixes are determined

$$[T_{AB}],[T_{BC}],[T_{CD}],[H_{AB}],[H_{BC}],[H_{CD}] \quad (28)$$

for each value, from degree to degree of angle $[\theta_1]$, by using the relations (7) - (9);

- the following matrixes are determined

$$\{U_B\},\{U_C\},\{U_D\},[\tilde{U}_B],[\tilde{U}_C],[\tilde{U}_D],[U],[\tilde{U}] \quad (29)$$

by using the relations (10) - (13);

- the displacement matrix $\{\xi\}$ from the cinematic couples is determined with relation (20) and the reaction matrix $\{F_A\}$ is determined with the relation (21);
- the reactions $\{F_B\},\{F_C\},\{F_D\}$, are determined by using the relations (22);
- the reactions variation graph is charted.

IV. NUMERICAL APLICATION

The bars from Fig. 1., $AA', A'A'', A''B, BO$, $OC, CD'', D''D', D'D$ are attributed, respectively the indexes $i=1,2,\ldots 8$, and considered that have the lengths l_i, diameters d_i and that are made from the same material with the elasticity module E, G, so:

$l_i = l = 0{,}04m$; $i=1,2,\ldots 8$; $d_i = d = 0{,}03m$; $i=1,2,\ldots 8$;

$E = 2{,}1\cdot 10^{11} daN/m$; $G = 2{,}1\cdot 10^{10} daN/m$; $\tilde{M} = 1Nm$

To solve this problem may use any conventional method of calculation numerical.

On the basis of a calculation program realized in Excel with the algorithm presented in the paper, where obtained, in the case where $\alpha = 0^\circ$, for the reactions $R^0_{AX}, R^0_{AY}, R^0_{AZ}$ and for the moments $M^0_{AX}, M^0_{AY}, M^0_{AZ}$, the results from diagram from Fig. 2. and Fig. 3. and for the reactions R_{AX}, R_{AY}, R_{AZ}, from the own reference system, the results are in the diagrams from Fig. 4. and Fig. 5.

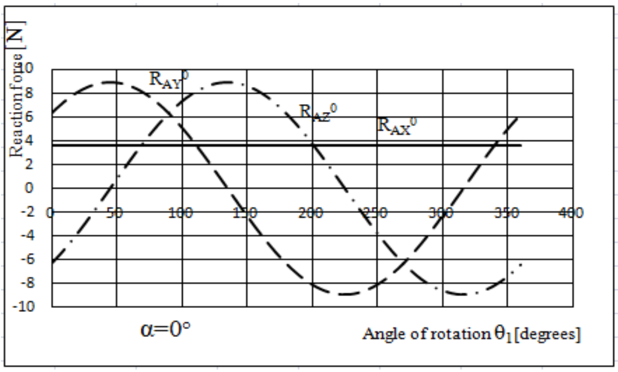

Fig. 2. The variation of reaction forces in general reference system in joint A, for $\alpha=0°$.

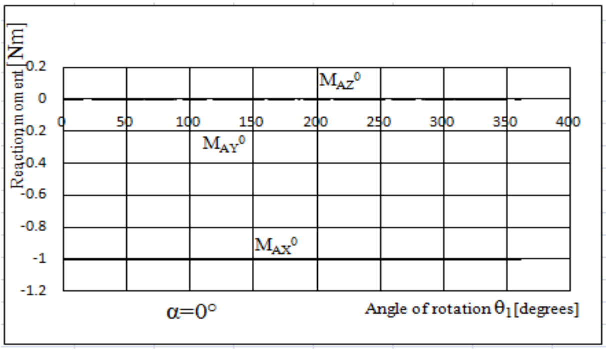

Fig. 3. The variation of reaction moments in general reference system in joint A, for $\alpha=0°$.

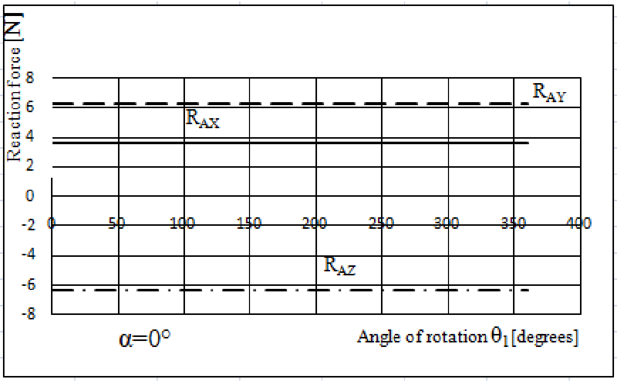

Fig. 4. The variation of reaction forces in local reference system in joint A, for $\alpha=0°$.

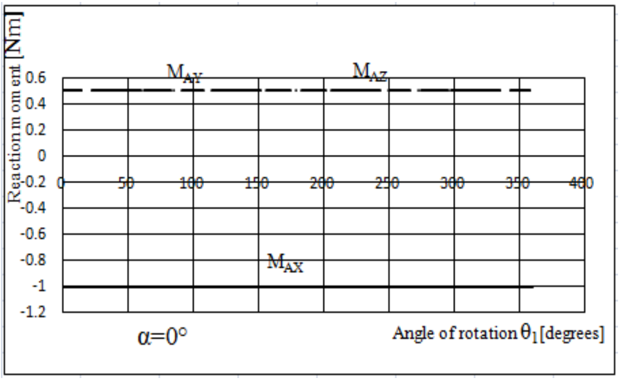

Fig. 5. The variation of reaction moments in local reference system in joint A, for $\alpha=0°$.

It is found as expected that in the local reference systems the reactions, forces of moments, are constant and this is due to the invariable positioning towards these systems.

Analogue, for the reaction from the other joints, in the own reference systems, are obtained constant values presented in Fig. 6. and Fig. 7.

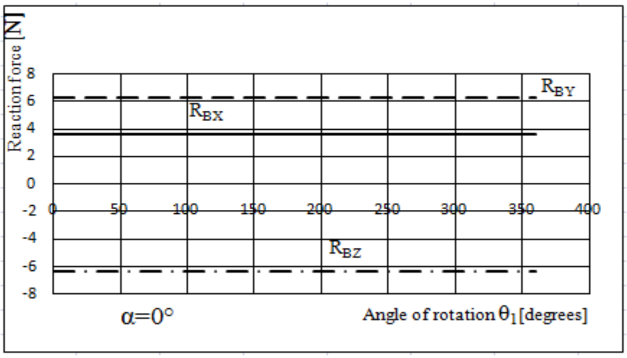

Fig. 6. The variation of reaction forces in local reference system in joint B, for *α=0°*.

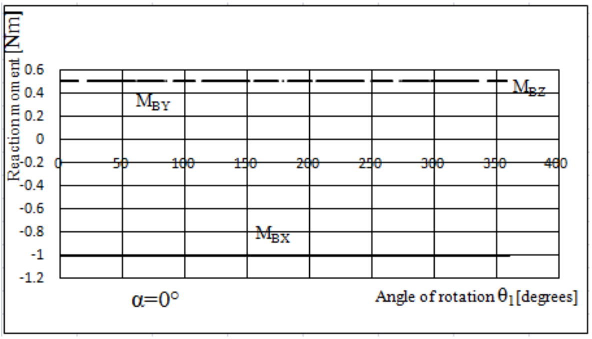

Fig. 7. The variation of reaction moments in local reference system in joint B, for *α=0°*.

For the case where $\alpha = 10^\circ$ are obtained the reactions $R_{AX}^0, R_{AY}^0, R_{AZ}^0$, $M_{AX}^0, M_{AY}^0, M_{AZ}^0$, and the representations are in Fig. 8. and Fig. 9.

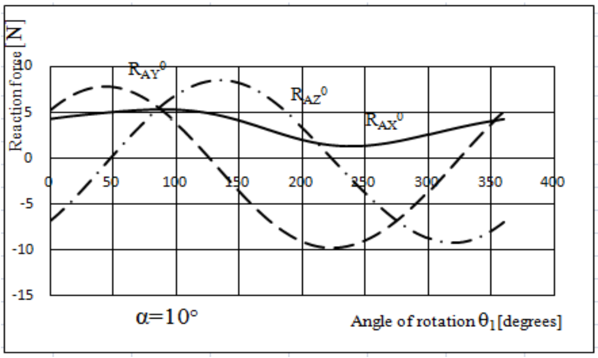

Fig. 8. The variation of reaction forces in general reference system in joint A, for *α=10°*.

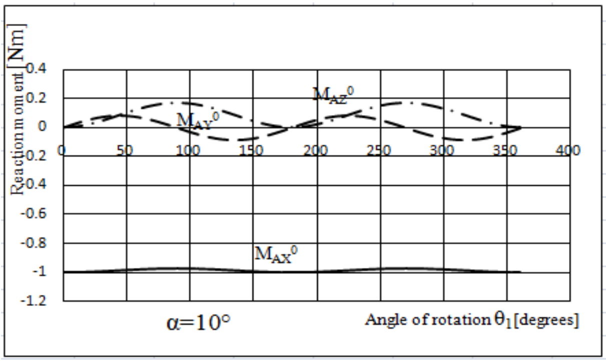

Fig. 9. The variation of reaction moments in general reference system in joint A, for *α=10°*.

It is found that the reaction R_{AX}^0 varies with the angle θ_1, and the reaction moments with the variations described by the relations (11).

In the own reference system, the components $R_{AX}^0, R_{AY}^0, R_{AZ}^0$, $M_{AX}^0, M_{AY}^0, M_{AZ}^0$, aren't constant and have a variation depending on the angle θ_1, as described in the diagrams from Fig. 10. and Fig. 11.

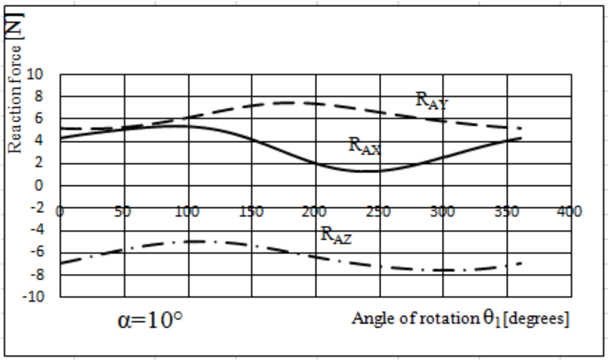

Fig. 10. The variation of reaction forces in local reference system in joint A, for *α=10°*.

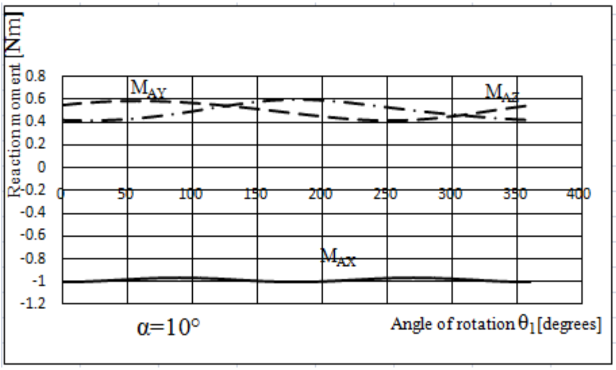

Fig. 11. The variation of reaction moments in local reference system in joint A, for *α=10°*.

For the reactions from the joint B, the representations in the own reference system are obtained in the diagrams from Fig. 12. and Fig. 13.

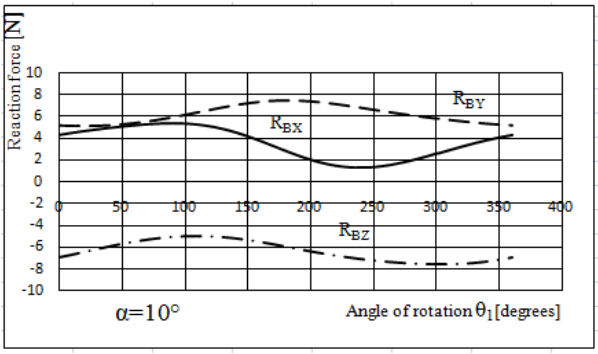

Fig. 12. The variation of reaction forces in local reference system in joint B, for *α=10°*.

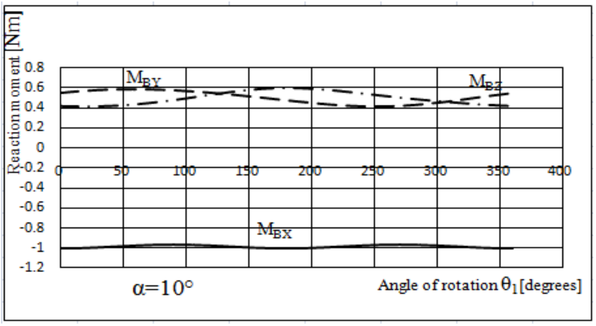

Fig. 13. The variation of reaction moments in local reference system in joint B, for *α=10°*.

Then, when the angle α increases, $\alpha = 20^\circ, \alpha = 40^\circ$, the solution for the components $R^0_{AX}, R^0_{AY}, R^0_{AZ}$, $M^0_{AX}, M^0_{AY}, M^0_{AZ}$, are represented in Fig. 14., Fig. 15., Fig. 16. and Fig. 17.

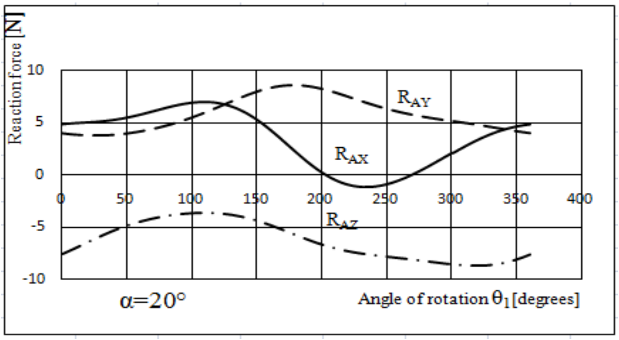

Fig. 14. The variation of reaction forces in local reference system in joint A, for *α=20°*.

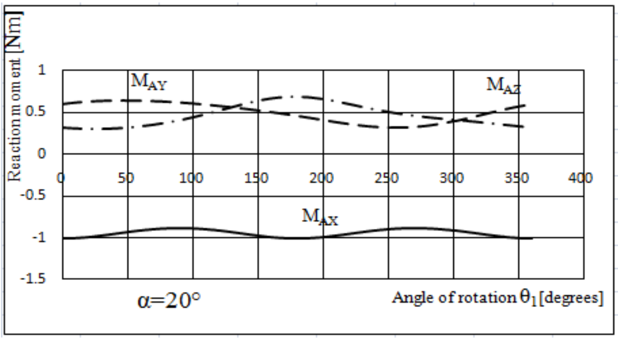

Fig. 15. The variation of reaction moments in local reference system in joint A, for *α=20°*.

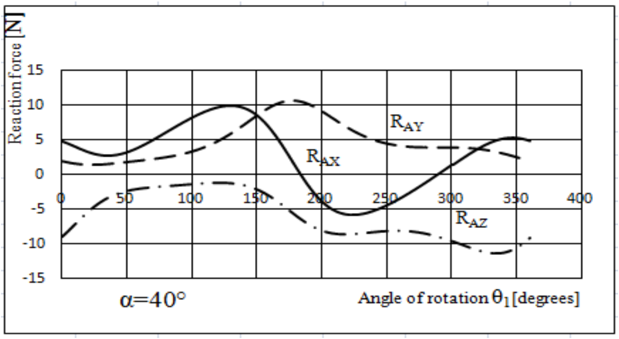

Fig. 16. The variation of reaction forces in local reference system in joint A, for *α=40°*.

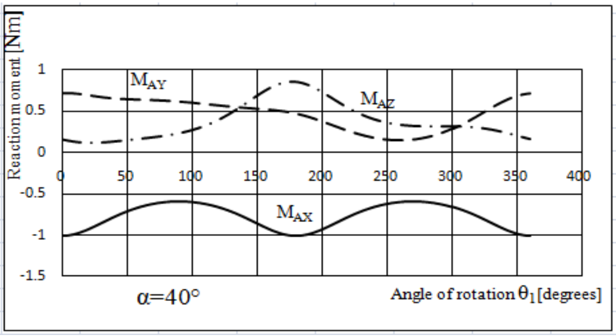

Fig. 17. The variation of reaction moments in local reference system in joint A, for *α=40°*.

It is noticed that the variation period of the axial reaction R_{AX} increases from 3,6N ($\alpha = 0^\circ$) to 4N ($\alpha = 10^\circ$), to 7N ($\alpha = 20^\circ$) and to 15N ($\alpha = 40^\circ$), the reaction having also negative values.

V. CONCLUSIONS

Following the numerical simulations made with the program realized using the algorithm from the present paper, the following conclusions can be taken.

For α=0°, in the general reference system $OX_0Y_0Z_0$ the reaction forces and moments vary with the angle θ_1, and in the local reference system they remain constant, depending only on the mechanical and geometrical characteristics of the element.

For α≠0°, both in local and general reference systems, the reactions and moments vary with the angle θ_1.

The forces and moments from the reaction are increasing as the angle of inclination α increases.

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